**Lecture 12-13.**

**Local Extreme Values. The first derivative test. The second derivative test. Using the highest derivatives**

**Endpoint Extreme Values. Absolute Extreme Values.**

**Increase and decrease of functions.** The function $y=f(x)$ is called *increasing* on some interval if, for any points $x\_{1}$ and $x\_{2}$ which belong to this interval, from the inequality $x\_{1}<x\_{2}$ we get the inequality $f\left(x\_{2}\right)<f(x\_{2})$.

The function $y=f(x)$ is called *decreasing* on some interval if, for any points $x\_{1}$ and $x\_{2}$ which belong to this interval, from the inequality $x\_{1}<x\_{2}$ we get the inequality $f\left(x\_{2}\right)>f(x\_{2})$.

If $y=f(x)$ is continuous on the interval $\left[a,b\right]$ and $f^{'}\left(x\right)>0$ for $a<x<b,$ then $f(x)$ increases on the interval $\left[a,b\right].$

If $y=f(x)$ is continuous on the interval $\left[a,b\right]$ and $f^{'}\left(x\right)<0$ for $a<x<b,$ then $f(x)$ decreases on the interval $\left[a,b\right].$

In the simplest cases, the domain of definition of $f(x)$ may be subdivided into a finite number of intervals of increase and decrease of the function ( intervals of monotonicity). These intervals are bounded by critical points $x$ (where $f^{'}\left(x\right)=0$ or $f^{'}\left(x\right)$ does not exist).

**Extremum of a function.** If there exists a two-sided neighborhood of a point $x\_{0}$ such that $∀x\ne x\_{0}$ of this neighborhood we have the inequality $f\left(x\right)>f(x\_{0})$, then the point $x\_{0}$ is called the *minimum point* *of the function*. Similarly, if for any point $∀x\ne x\_{0}$ the inequality $f\left(x\right)<f(x\_{0})$ is fulfilled then the point $x\_{0}$ is called the *maximum point of the function.* The minimum point or maximum point of a function is its *extremal point.*

If $x\_{0}$ is an extremal point of the function $f\left(x\right),$ then $f^{'}\left(x\right)=0,$ or $f^{'}\left(x\right)$ does not exist (*necessary condition for the existence of an extremum*).

The *sufficient conditions* *for the existence and absence* *of an extremum* of a continuous function $f\left(x\right)$ are given by the following rules:

1. If there exists a neighborhood $(x\_{0}-δ,x\_{0}+δ)$ of a critical point $x\_{0}$ such that $f^{'}\left(x\right)>0$ for $x\_{0}-δ<x<x\_{0}$ and $f^{'}\left(x\right)<0$ for $x\_{0}<x<x\_{0}+δ$, then $x\_{0}$ is maximum point of the function $f\left(x\right);$ and if $f^{'}\left(x\right)<0$ for $x\_{0}-δ<x<x\_{0}$ and $f^{'}\left(x\right)>0$ for $x\_{0}<x<x\_{0}+δ$, then $x\_{0}$ is minimum point of the function $f\left(x\right).$
2. If $f^{'}\left(x\_{0}\right)=0$ and $f^{'}'\left(x\_{0}\right)<0$, then $x\_{0}$ is maximum point; if $f^{'}\left(x\_{0}\right)=0$ and $f^{'}'\left(x\_{0}\right)>0$, then $x\_{0}$ is minimum point; but if $f^{'}\left(x\_{0}\right)=0,$

$f^{'}'\left(x\_{0}\right)=0,$ $f^{'}''\left(x\_{0}\right)\ne 0$, then the point $x\_{0}$ is not an extremal point of the function $f\left(x\right).$

More generally: let the first of the derivatives (not equal to zero at the point $x\_{0})$ of the function $f\left(x\right)$ be of the order $k$. Then, if $k$ is even, the point $x\_{0}$ is not an extremal point, namely, the maximum point if $f^{\left(k\right)}\left(x\_{0}\right)<0$; and it is the minimum point if $f^{\left(k\right)}\left(x\_{0}\right)>0.$ But if $k$ is odd, then $x\_{0}$ is not an extremal point.